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A note on the angular momentum analysis $SU(2l+1) \supset O^+(2l+1) \supset O^+(3)$

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Abstract. A method for carrying out the angular momentum analysis

$$SU(2l+1) \supset O^+(2l+1) \supset O^+(3)$$

is discussed using Littlewood's S function technique. Results for $l = 4$ are tabulated.

1. Introduction

Syamala Devi (1972) discussed the problem of finding the $O^+(3)$ content of an irreducible representation (IR) of $SU(2l+1)$ using Littlewood's (1944) 'new multiplication' of S functions. A method of carrying out the angular momentum analysis

$$SU(2l+1) \supset O^+(2l+1) \supset O^+(3)$$

was discussed in Hamermesh (1962) and explicit results are tabulated for the cases $l = 2$ and $l = 3$. Here we discuss the same angular momentum analysis by using S functions and their 'new multiplication', known as the 'plethysm operation'.

2. Formulation of the method

Using theorem II of Littlewood (1950) we first find the $O^+(2l+1)$ content of an IR of $SU(2l+1)$. Let $[\lambda]$ by any such IR of $O^+(2l+1)$ contained in an IR of $SU(2l+1)$. We then express $[\lambda]$ in terms of the S functions using theorem I of Littlewood (1950):

$$[\lambda] = \{\lambda\} + \sum (-1)^{p/2} g_{\gamma\eta\lambda}\{\eta\}, \tag{1}$$

summed for all S functions of the set $\{\gamma\}$ such that $\{\lambda\}$ appears in a product $\{\gamma\}\ \{\eta\}$ with coefficient $g_{\gamma\eta\lambda}$, (γ) being a partition of p. The $O^+(3)$ content of each IR of $SU(2l+1)$ appearing in the right-hand side of (1) is then found by calculating the plethysms $\{2l\}\otimes\{\lambda\}$ and the various $\{2l\}\otimes\{\eta\}$. There are various methods of calculating the plethysms. Out of these we find the following formula (Littlewood 1958) as a more straightforward one, well suited to our present needs:

$$\{n\}\otimes\{\lambda\} = \prod_{i<j=2}^{n+1} \left(\frac{p^{\lambda_i+j} - p^{\lambda_j+i}}{p^j - p^i}\right). \tag{2}$$

Taking the algebraic sum of these $O^+(3)$ contents, coefficients being prescribed by the right-hand side of (1), we get the $O^+(3)$ content of the IR $[\lambda]$ of $O^+(2l+1)$. Though this method is quite general in nature, actual calculations of the pethysms will become laborious when we go to higher l values. As far as we know the literature does not contain the case of $l = 4$. We therefore give the angular momentum analysis $SU(9) \supset O^+(9) \supset O^+(3)$ in table 1. The modification rules required in the table are taken from Newell (1951). Dots (...) are used in the $O^+(3)$ column against some IRs of $O^+(9)$ to indicate that those $O^+(3)$ contents are already specified above in the table.

Table 1. Angular momentum analysis $SU(9) \supset O^+(9) \supset O^+(3)$

r	SU(9)	$O^+(9)$	$O^+(3)$
0	{0}	[0]	0
1	{1}	[1]	4
2	{2}	[2]	8, 6, 4, 2
		[0]	...
	$\{1^2\}$	$[1^2]$	7, 5, 3, 1
3	{2 1}	[2 1]	$11, 10, 9, 8^2, 7^2, 6^2, 5^3, 4^2, 3^2, 2^2, 1$
		[1 0]	...
	$\{1^3\}$	$[1^3]$	$9, 7, 6, 5, 4, 3^2, 1$
4	$\{2^2\}$	$[2^2]$	$14, 12^2, 11, 10^3, 9^2, 8^4, 7^3, 6^5, 5^3, 4^5, 3^2, 2^4, 0^2$
		[2 0]	...
		[0]	...
	$\{2\ 1^2\}$	$[2\ 1^2]$	$13, 12, 11^2, 10^2, 9^4, 8^4, 7^5, 6^5, 5^6, 4^5, 3^5, 2^3, 1^3$
		$[1^2]$	...
	$\{1^4\}$	$[1^4]$	$10, 8, 7, 6^2, 5, 4^2, 3, 2^2, 0$
5	$\{2^2\ 1\}$	$[2^2\ 1]$	$16, 15, 14^2, 13^3, 12^5, 11^5, 10^8, 9^9, 8^{11}, 7^{11}, 6^{13}, 5^{11}, 4^{12}, 3^9, 2^8, 1^4, 0^3$
		[2 1]	...
		[1]	...
	$\{2\ 1^3\}$	$[2\ 1^3]$	$14, 13, 12^2, 11^3, 10^4, 9^5, 8^7, 7^7, 6^8, 5^8, 4^8, 3^6, 2^6, 1^3, 0$
		$[1^3]$	...
	$\{1^5\} \equiv \{1^4\}$	$[1^4]$	...
6	$\{2^3\}$	$[2^3]$	$18, 16^2, 15^2, 14^3, 13^4, 12^7, 11^6, 10^{10}, 9^{10}, 8^{12}, 7^{11}, 6^{15}, 5^{10}, 4^{13}, 3^9, 2^8, 1^3, 0^4$
		$[2^2]$	...
		[2]	...
		[0]	...
	$\{2^2\ 1^2\}$	$[2^2\ 1^2]$	$17, 16, 15^3, 14^3, 13^6, 12^7, 11^{11}, 10^{12}, 9^{16}, 8^{16}, 7^{20}, 6^{18}, 5^{21}, 4^{16}, 3^{17}, 2^{10}, 1^9, 0$
		$[2\ 1^2]$	...
		$[1^2]$	...
	$\{2\ 1^4\}$	$[2\ 1^4] \equiv [2\ 1^3]$	...
		$[1^4]$	...
	$\{1^6\} \equiv \{1^3\}$	$[1^3]$	...
7	$\{2^3\ 1\}$	$[2^3\ 1]$	$19, 18, 17^2, 16^3, 15^5, 14^7, 13^{10}, 12^{12}, 11^{16}, 10^{18}, 9^{22}, 8^{23}, 7^{26}, 6^{25}, 5^{25}, 4^{22}, 3^{20}, 2^{14}, 1^{10}, 0^2$
		$[2^2\ 1]$	...
		[2 1]	...
		[1]	...
	$\{2^2\ 1^3\}$	$[2^2\ 1^3] \equiv [2^2\ 1^2]$	...
		$[2\ 1^3]$	...
		$[1^3]$	...
	$\{2\ 1^5\}$	$[2\ 1^5] \equiv [2\ 1^2]$	...
		$[1^5] \equiv [1^4]$	...
	$\{1^7\} \equiv \{1^2\}$	$[1^2]$	...

Table 1—*continued*

r	SU(9)	$O^+(9)$	$O^+(3)$
8	$\{2^4\}$	$[2^4]$	$20, 18, 17^2, 16^3, 15^3, 14^6, 13^6, 12^9, 11^{10}, 10^{13}, 9^{12}, 8^{17}, 7^{14}, 6^{16}, 5^{15}, 4^{15}, 3^9, 2^{11}, 1^4, 0^3$
		$[2^3]$	...
		$[2^2]$	...
		$[2]$	...
		$[0]$	...
	$\{2^3\,1^2\}$	$[2^3\,1^2] \equiv [2^3\,1]$	...
		$[2^2\,1^2]$	...
		$[2\,1^2]$	...
		$[1^2]$	...
	$\{2^2\,1^4\}$	$[2^2\,1^4] \equiv [2^2\,1]$	...
		$[2\,1^4] \equiv [2\,1^3]$	...
		$[1^4]$	...
	$\{2\,1^6\}$	$[2\,1^6] \equiv [2\,1]$	...
		$[1^6] \equiv [1^3]$	...
	$\{1^8\} \equiv \{1\}$	$[1]$	...
9	$\{2^4\,1\}$	$[2^4\,1] \equiv [2^4]$	...
		$[2^3\,1]$	...
		$[2^2\,1]$	...
		$[2\,1]$	...
		$[1]$	...
	$\{2^3\,1^3\}$	$[2^3\,1^3] \equiv [2^3]$	...
		$[2^2\,1^3] \equiv [2^2\,1^2]$	...
		$[2\,1^3]$	...
		$[1^3]$	...
	$\{2^2\,1^5\}$	$[2^2\,1^5] \equiv [2^2]$	...
		$[2\,1^5] \equiv [2\,1^2]$	...
		$[1^5] \equiv [1^4]$	...
	$\{2\,1^7\}$	$[2\,1^7] \equiv [2]$	...
		$[1^7] \equiv [1^2]$	...
	$\{1^9\} \equiv \{0\}$	$[0]$	...

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